

2017

AP[®]

CollegeBoard

AP Calculus AB

Scoring Guidelines

AP[®] CALCULUS AB/CALCULUS BC
2017 SCORING GUIDELINES

Question 1

	1 : units in parts (a), (c), and (d)
(a) $\text{Volume} = \int_0^{10} A(h) \, dh$ $\approx (2 - 0) \cdot A(0) + (5 - 2) \cdot A(2) + (10 - 5) \cdot A(5)$ $= 2 \cdot 50.3 + 3 \cdot 14.4 + 5 \cdot 6.5$ $= 176.3$ cubic feet	2 : $\begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \end{cases}$
(b) The approximation in part (a) is an overestimate because a left Riemann sum is used and A is decreasing.	1 : overestimate with reason
(c) $\int_0^{10} f(h) \, dh = 101.325338$ The volume is 101.325 cubic feet.	2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) Using the model, $V(h) = \int_0^h f(x) \, dx$. $\left. \frac{dV}{dt} \right _{h=5} = \left[\frac{dV}{dh} \cdot \frac{dh}{dt} \right]_{h=5}$ $= \left[f(h) \cdot \frac{dh}{dt} \right]_{h=5}$ $= f(5) \cdot 0.26 = 1.694419$ When $h = 5$, the volume of water is changing at a rate of 1.694 cubic feet per minute.	3 : $\begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

AP[®] CALCULUS AB
2017 SCORING GUIDELINES

Question 2

(a) $\int_0^2 f(t) \, dt = 20.051175$

20.051 pounds of bananas are removed from the display table during the first 2 hours the store is open.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $f'(7) = -8.120$ (or -8.119)

After the store has been open 7 hours, the rate at which bananas are being removed from the display table is decreasing by 8.120 (or 8.119) pounds per hour per hour.

2 : $\begin{cases} 1 : \text{value} \\ 1 : \text{meaning} \end{cases}$

(c) $g(5) - f(5) = -2.263103 < 0$

Because $g(5) - f(5) < 0$, the number of pounds of bananas on the display table is decreasing at time $t = 5$.

2 : $\begin{cases} 1 : \text{considers } f(5) \text{ and } g(5) \\ 1 : \text{answer with reason} \end{cases}$

(d) $50 + \int_3^8 g(t) \, dt - \int_0^8 f(t) \, dt = 23.347396$

23.347 pounds of bananas are on the display table at time $t = 8$.

3 : $\begin{cases} 2 : \text{integrals} \\ 1 : \text{answer} \end{cases}$

AP[®] CALCULUS AB/CALCULUS BC
2017 SCORING GUIDELINES

Question 3

(a) $f(-6) = f(-2) + \int_{-2}^{-6} f'(x) \, dx = 7 - \int_{-6}^{-2} f'(x) \, dx = 7 - 4 = 3$

$$f(5) = f(-2) + \int_{-2}^5 f'(x) \, dx = 7 - 2\pi + 3 = 10 - 2\pi$$

$$3 : \begin{cases} 1 : \text{uses initial condition} \\ 1 : f(-6) \\ 1 : f(5) \end{cases}$$

(b) $f'(x) > 0$ on the intervals $[-6, -2]$ and $(2, 5]$.

Therefore, f is increasing on the intervals $[-6, -2]$ and $[2, 5]$.

2 : answer with justification

(c) The absolute minimum will occur at a critical point where $f'(x) = 0$ or at an endpoint.

$$f'(x) = 0 \Rightarrow x = -2, x = 2$$

x	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

$$2 : \begin{cases} 1 : \text{considers } x = 2 \\ 1 : \text{answer with justification} \end{cases}$$

The absolute minimum value is $f(2) = 7 - 2\pi$.

(d) $f''(-5) = \frac{2 - 0}{-6 - (-2)} = -\frac{1}{2}$

$$2 : \begin{cases} 1 : f''(-5) \\ 1 : f''(3) \text{ does not exist,} \\ \text{with explanation} \end{cases}$$

$$\lim_{x \rightarrow 3^-} \frac{f'(x) - f'(3)}{x - 3} = 2 \text{ and } \lim_{x \rightarrow 3^+} \frac{f'(x) - f'(3)}{x - 3} = -1$$

$f''(3)$ does not exist because

$$\lim_{x \rightarrow 3^-} \frac{f'(x) - f'(3)}{x - 3} \neq \lim_{x \rightarrow 3^+} \frac{f'(x) - f'(3)}{x - 3}.$$

AP[®] CALCULUS AB/CALCULUS BC
2017 SCORING GUIDELINES

Question 4

(a) $H'(0) = -\frac{1}{4}(91 - 27) = -16$
 $H(0) = 91$

An equation for the tangent line is $y = 91 - 16t$.

The internal temperature of the potato at time $t = 3$ minutes is approximately $91 - 16 \cdot 3 = 43$ degrees Celsius.

(b) $\frac{d^2H}{dt^2} = -\frac{1}{4} \frac{dH}{dt} = \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)(H - 27) = \frac{1}{16}(H - 27)$

$$H > 27 \text{ for } t > 0 \Rightarrow \frac{d^2H}{dt^2} = \frac{1}{16}(H - 27) > 0 \text{ for } t > 0$$

Therefore, the graph of H is concave up for $t > 0$. Thus, the answer in part (a) is an underestimate.

(c) $\frac{dG}{(G - 27)^{2/3}} = -dt$

$$\int \frac{dG}{(G - 27)^{2/3}} = \int (-1) dt$$

$$3(G - 27)^{1/3} = -t + C$$

$$3(91 - 27)^{1/3} = 0 + C \Rightarrow C = 12$$

$$3(G - 27)^{1/3} = 12 - t$$

$$G(t) = 27 + \left(\frac{12 - t}{3}\right)^3 \text{ for } 0 \leq t < 10$$

The internal temperature of the potato at time $t = 3$ minutes is

$$27 + \left(\frac{12 - 3}{3}\right)^3 = 54 \text{ degrees Celsius.}$$

3 : $\begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line} \\ 1 : \text{approximation} \end{cases}$

1 : underestimate with reason

5 : $\begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration and} \\ \quad \text{uses initial condition} \\ 1 : \text{equation involving } G \text{ and } t \\ 1 : G(t) \text{ and } G(3) \end{cases}$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

AP[®] CALCULUS AB
2017 SCORING GUIDELINES

Question 5

(a) $x'_P(t) = \frac{2t-2}{t^2-2t+10} = \frac{2(t-1)}{t^2-2t+10}$

$t^2 - 2t + 10 > 0$ for all t .

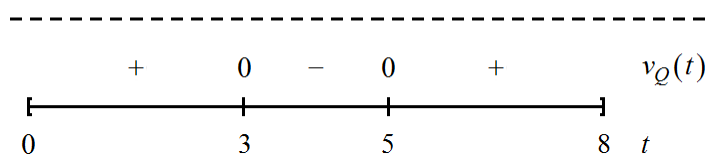
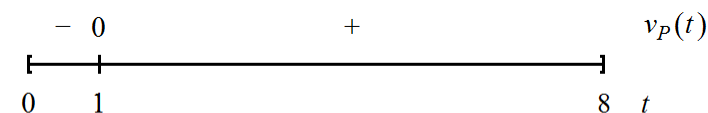
$x'_P(t) = 0 \Rightarrow t = 1$

$x'_P(t) < 0$ for $0 \leq t < 1$.

Therefore, the particle is moving to the left for $0 \leq t < 1$.

(b) $v_Q(t) = (t-5)(t-3)$

$v_Q(t) = 0 \Rightarrow t = 3, t = 5$



Both particles move in the same direction for $1 < t < 3$ and $5 < t \leq 8$ since $v_P(t) = x'_P(t)$ and $v_Q(t)$ have the same sign on these intervals.

(c) $a_Q(t) = v'_Q(t) = 2t - 8$

$a_Q(2) = 2 \cdot 2 - 8 = -4$

$a_Q(2) < 0$ and $v_Q(2) = 3 > 0$

At time $t = 2$, the speed of the particle is decreasing because velocity and acceleration have opposite signs.

(d) Particle Q first changes direction at time $t = 3$.

$$\begin{aligned} x_Q(3) &= x_Q(0) + \int_0^3 v_Q(t) dt = 5 + \int_0^3 (t^2 - 8t + 15) dt \\ &= 5 + \left[\frac{1}{3}t^3 - 4t^2 + 15t \right]_{t=0}^{t=3} = 5 + (9 - 36 + 45) = 23 \end{aligned}$$

2 : $\begin{cases} 1 : x'_P(t) \\ 1 : \text{interval} \end{cases}$

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{analysis using } v_P(t) \text{ and } v_Q(t) \end{cases}$

Note: 1/2 if only one interval with analysis

Note: 0/2 if no analysis

2 : $\begin{cases} 1 : a_Q(2) \\ 1 : \text{speed decreasing with reason} \end{cases}$

3 : $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

AP[®] CALCULUS AB
2017 SCORING GUIDELINES

Question 6

(a) $f'(x) = -2\sin(2x) + \cos x e^{\sin x}$

$$f'(\pi) = -2\sin(2\pi) + \cos \pi e^{\sin \pi} = -1$$

2 : $f'(\pi)$

(b) $k'(x) = h'(f(x)) \cdot f'(x)$

$$\begin{aligned} k'(\pi) &= h'(f(\pi)) \cdot f'(\pi) = h'(2) \cdot (-1) \\ &= \left(-\frac{1}{3}\right)(-1) = \frac{1}{3} \end{aligned}$$

2 : $\begin{cases} 1 : k'(x) \\ 1 : k'(\pi) \end{cases}$

(c) $m'(x) = -2g'(-2x) \cdot h(x) + g(-2x) \cdot h'(x)$

$$\begin{aligned} m'(2) &= -2g'(-4) \cdot h(2) + g(-4) \cdot h'(2) \\ &= -2(-1)\left(-\frac{2}{3}\right) + 5\left(-\frac{1}{3}\right) = -3 \end{aligned}$$

3 : $\begin{cases} 2 : m'(x) \\ 1 : m'(2) \end{cases}$

(d) g is differentiable. $\Rightarrow g$ is continuous on the interval $[-5, -3]$.

$$\frac{g(-3) - g(-5)}{-3 - (-5)} = \frac{2 - 10}{2} = -4$$

Therefore, by the Mean Value Theorem, there is at least one value c , $-5 < c < -3$, such that $g'(c) = -4$.

2 : $\begin{cases} 1 : \frac{g(-3) - g(-5)}{-3 - (-5)} \\ 1 : \text{justification,} \\ \quad \text{using Mean Value Theorem} \end{cases}$